Exam Lie Groups in Physics

Date	November 4, 2013
Room	V 5161.0289
Time	14:00 - 17:00
Lecturer	D. Boer

- Write your name and student number on every separate sheet of paper
- Raise your hand for more paper
- You are not allowed to use the lecture notes, nor other notes or books
- The weights of the three exercises are given below
- Answers may be given in Dutch
- Illegible handwriting will be graded as incorrect
- Good luck!

Weighting

1a)	12	2a)	10	3a)	8
1b)	12	2b)	10	3b)	10
1c)	10	2c)	8		
1d)	10				

$$\text{Result} = \frac{\sum \text{points}}{10} + 1$$

Exercise 1
Consider the group $O(1,1)$ defined by 2×2 matrices O satisfying

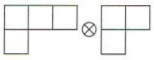
$$O^T = gO^{-1}g^{-1} \quad \text{with} \quad g = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(a) Write down the general form of elements O in $O(1,1)$ and show that such matrices form a non-compact non-Abelian group.

(b) Specify the connected components of $O(1,1)$ and show that they form cosets of the connected subgroup. Describe the corresponding factor group.

(c) Show whether the defining representation is irreducible.

(d) Write down the corresponding representation of the Lie algebra of $O(1,1)$ and show whether it is an irrep of the Lie algebra.

Exercise 2
(a) Decompose the direct product of irreps of $su(n)$ given by  into irreps.

(b) Count the dimensions of the irreps for $su(2)$ and $su(3)$ by using the hooks factors. Indicate complex conjugate irreps whenever appropriate.

(c) Relate the decomposition for $su(2)$ to the corresponding case of addition of angular momentum in Quantum Mechanics.

Exercise 3
Consider the four-dimensional representation of the generators of the Lorentz group:

$$(M^{\mu\nu})^{\alpha}{}_{\beta} = i(g^{\mu\alpha}g^{\nu}{}_{\beta} - g^{\nu\alpha}g^{\mu}{}_{\beta})$$

(a) Write down the matrices for the following two cases: $\mu = 0, \nu = 1$ and $\mu = 2, \nu = 3$.

(b) Exponentiate the matrix M^{01} obtained in part (a) and explain which Lorentz transformation it corresponds to.